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SHEAR-LAG TESTS OF TWO BOX BEAMS

WITH FLAT COVERS LOADED TO DESTRUCTION

By Patrick T. Chiarito

Langley Memorial Aeronautical Laboratory
Langley Field, Va.

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SUMMARY

Strain-gage tests were made on two box beams loaded to destruction in an attempt to verify the shear-lag theory at stresses beyond the yield point. The test results indicated that the corner-flange stresses can be predicted with a fair degree of accuracy. Collapse of both beams was precipitated by failure of the corner angles at stresses close to the column yield stress of the material.

INTRODUCTION

The engineering theory of bending is sufficiently accurate for defining the stresses in all the fibers of a prismatic beam with a solid rectangular cross section of reasonable depth-width ratio. A discrepancy appears when the beam is made from thin material and the cross section consists of distinct webs and flanges. The discrepancy becomes more pronounced when the flange is both wide and thin, as is common in airplane structures.

This deviation from the engineering bending theory may exist in both the tension and compression flanges. Theories that have come to be known as "shear-lag" theories have been developed to take the deviation into account. A shear-lag theory appears in reference 1 with methods of analysis and experimental verification.

Heretofore experimental investigations have been confined mainly to the study of shear lag at low stresses. In the present investigation, an open box beam was tested to failure while strain measurements were taken near the root. The beam was rebuilt, and another ultimate-strength test followed. In this paper there are presented comparisons between calculated and experimental stresses for the two test beams. All the analyses were made in accordance with the methods of reference 1.

Details of the analysis of the first beam are given in the appendix.

BENDING TESTS OF THE BOX BEAMS

Specimens and Apparatus

An open box beam was tested to destruction, with the cover on the compression side. The beam was rebuilt and another test to destruction was made. The original beam and the rebuilt beam will be referred to as beam 1 and beam 2, respectively.

The cover was designed to have a small ratio of the area of the corner-flange angle to the area of the longitudinal stringers in order to yield a larger shear-lag effect than is usually found in actual structures. This design provides a rather severe check on the theory.

Test specimens.- Because the wing structure is the part of an airplane most affected by shear lag, the specimens for this investigation were built similar to an actual wing. Details of the test beams are shown in figures 1 and 2. Both beams were made from 24S-T aluminum alloy, except for transverse bulkheads, which were made of steel. These bulkheads were flanged along three sides for attachment to the shear webs and stringers.

For the first series of tests the box was of constant cross section throughout. After failure the corner flange was reinforced for a length of 22 inches on either side of the root by another angle formed from 0.064-inch sheet and the second series of tests was performed. When the box was being rebuilt, the cover was moved 1½ bays longitudinally, and the surplus at one end was cut off and spliced to the other end. Thus, the previously damaged part of the cover was moved to a point of comparatively low stress. New corner-flange angles were used.

The full span, with symmetry about both the longitudinal and transverse axes, was used in order to obtain the closest possible approximation to a fixed root. Local variations in the material and accidental eccentricities in construction and test loading still present an obstacle to the ideal fixed-root condition.

Apparatus.- Details of the apparatus are shown in figure 3. In order to facilitate the actual loading operation, the double whippletree was used to anchor the beam to the floor by the steel straps at the bulkheads. The load was applied at the center of the full span by a portable hydraulic jack of 100 kips capacity.

Procedure

Loading conditions.- Two conditions of loading were used: (1) one concentrated load at the tip of each shear web and (2) four concentrated loads equally spaced along each web of the half span. This distributed-load condition was used for the tests that were carried to failure because it approximates actual loading and also produces a larger shear-lag effect for a given bending moment at the root.

Method of obtaining data.- Strain measurements were taken on the four quadrants of the cover of the full span near the root. As shown in figure 4, strain gages were mounted both on the outside of the cover and on the inside leg of the Z-stringers. At the root station no gages were used on the inside because of interference by the bulkhead.

All strain measurements were made with resistance-type electrical strain gages. There were approximately 135 strain gages used on each beam.

Strain was measured at a minimum of three load readings in the elastic range for each test in order to check the linear variation of stress with respect to applied load. In order to reduce thermal errors in the measured strains, a reasonable amount of control was exercised over the temperature in the vicinity of the test specimens.

Conversion of strain measurements to stresses.- The stress-strain curves for the materials of the cover (corner angle, sheet, and stringers) were obtained by the standard pack-compression method developed by the National Bureau of Standards (reference 2). Those curves were used for converting strain measurements to corresponding stresses.

Accuracy of measurements.- The total applied jack load was accurate to approximately 0.5 percent. The thicknesses of all parts made from flat sheet were obtained by micrometer measurements accurate to about ±0.0002 inch. The areas of the tension flange angles were taken from a structural aluminum handbook. Although the possible error in the areas of these angles was larger than for other parts of the beam cross section, the values were considered satisfactory. Strain measurements were made with an accuracy estimated to be ±4 percent.

SYMBOLS

A_F area of flange, square inches

A_L area of longitudinal, square inches

A_{st} area of idealized stringer, square inches

A_T total area of cover, square inches

E Young's modulus, pounds per square inch

G effective shear modulus, pounds per square inch

I geometric moment of inertia, inches^4

K shear-lag parameter

L length, inches

M bending moment, pound-inches

S shear force, pounds

X change in stringer force caused by shear deformation of cover sheet, pounds

Y auxiliary parameter (equation (14) of reference 1)

b width of half beam, inches

b_1 distance between rivet lines of adjacent stringers, inches

b_S width of substitute beam, inches

c distance from centroid to extreme fiber, inches

h effective depth of beam, inches

t thickness of cover sheet, inches

t_W thickness of shear web, inches

w effective width of sheet, inches

y distance from center line, inches

y_L distance from center line to resultant internal force, inches

γ shear strain

ρ radius of gyration, inches

σ_F normal stress in flange, pounds per square inch

σ_L normal stress in longitudinal, pounds per square inch

σ^P normal stress based on the assumption that plane cross sections remain plane (Mc/I), pounds per square inch

TEST RESULTS AND COMPARISONS WITH CALCULATIONS

Strain-Gage Tests

The strains measured on the outside surface of the cover differed greatly, at each gage station, from those measured on the inside legs of the Z-stringers. Factors that might have contributed to these large differences were: (1) secondary bending in the stringers and (2) inadequacy of the rivets for transmitting the required forces to the stringers.

The comparison between the experimental and calculated values was simplified by converting all stringer stresses to equivalent stresses at the centroid of the cover. This conversion was made by assuming linear variation between the stresses measured on the outside of the cover sheet and those measured on the inside legs of the Z-stringers.

<u>Beam 1 with distributed load</u>.—The chordwise distributions of stresses at several stations near the root are

shown in figure 5 for beam 1 with four concentrated loads. The data and calculated curves are shown for two values of load; namely, 8000 and 14,000 pounds jack load, or 500 and 875 pounds, respectively, at each loading strap. The lower load produced stress distributions that were typical of the elastic range; the higher load represented the highest load at which strain measurements were taken.

For the lower load the agreement between calculated values and test points is fair at the root. The curve does not fit the experimental points exactly. The corner-flange stresses agree very well, however, and the summation of the internal forces in the cover balances the external force M/h.

In the chordwise plots at stations $2\frac{1}{2}$ and 5 inches from the root, the low experimental values in the vicinity of the corner flanges indicated that the forces might not have entered the cover as predicted. This opinion was supported by the comparison between the internal and external forces. The internal forces were found to be 14 and 17 percent lower than the external forces at stations $2\frac{1}{2}$ and 5 inches, respectively. Because strains were not measured at every stringer across the entire width of the cover, the measured strains were used to estimate the strains that were lacking. These estimations may result in errors of approximately ±10 percent in total internal force. Inasmuch as the flange includes more area than any single stringer, a low value of stress at the flange has a noticeable influence on the total internal force.

The flange stresses were obtained from strain measurements taken on the cover sheet near the flange rivets. Deformations in the rivets, owing to failure of the rivets to fill the rivet holes completely, might have caused the forces to remain in the corner flange rather than to be transmitted to the adjoining sheet and stringers.

As shown in figure 4, measurements were taken on the cover sheet next to the flange. The experimental results for those gages should therefore be compared with values calculated for the outer fiber of the cover sheet. On the other hand, the weighted average of the measurements for each Z-stringer should be compared with the value calculated for the centroid of the cover. Thus, there is one calculated curve for each load at the root where strain measurements were taken only on the outside of the cover sheet, and two curves at the other stations where strains were measured on the Z-stringers as well as on the cover sheet.

For the higher load, 98 percent of ultimate, the corner-flange stresses at the root were not obtained. Figure 5(a) shows that the other values follow the trend of the data for the lower load with the exception of stringer 9. The unusually high stress in this stringer may be the result of a localized bending. (At failure, stringer 9 was disjoined from the root bulkhead.) This effect continues at station 2½ inches; the corresponding value at station 5 inches was not obtained.

Failure to obtain strain readings at the flange for the jack load of 14,000 pounds made it impracticable to compare internal and external forces. The weighted average of the stresses in the Z-stringers is shown for both loads. Agreement of this average with the calculated curve was good at 8000 pounds. It seems, however, that a different assumption for weighting should be used at 14,000 pounds. Too much weight was apparently given to the stress at the outside of the cover sheet at high stresses.

The two values of calculated stress that appear near the flanges - both for the stresses calculated without shear lag (Mc/I) and with shear lag - are for the centroid and the outer fiber of the cover.

<u>Beam 2 with one concentrated load at tip</u>.- Beam 2 was tested for checking the distribution of stresses for the tip-load condition without causing any of the material to yield; the maximum jack load was therefore restricted. The chordwise distribution of experimental stresses (fig. 6) at the root is in satisfactory agreement with the calculated values; the sum of the internal forces consequently checks with the external force.

<u>Beam 2 with distributed load</u>.- Figure 7 shows the chordwise distribution of stress at several stations for jack loads of 10,000 and 17,500 pounds or 625 and 1095 pounds per loading strap. As in the case for beam 1, the lower load on beam 2 gave typical results for the elastic range, and the higher load was the highest load at which strain measurements were taken. The agreement between calculated and experimental values is fair at the lower load. The strains that were measured near the heel of the outside corner angles at the root corresponded to stresses that were approximately 10 percent greater than the calcu-

lated stress for the root. The theory yields, however, a calculated stress in a fictitious corner flange of zero width. The flange actually has finite width and a chordwise variation in flange stress exists. The summation of the internal forces is approximately 7 percent greater than the external force at the root station and 5 percent lower at stations $2\frac{1}{2}$ and 5 inches.

For the jack load of 17,500 pounds, 98 percent of ultimate, the experimental values at the root agree very well with the calculated curve in the vicinity of the flanges. There are several "wild" points on the cover, which may indicate local disturbances. At stations $2\frac{1}{2}$ and 5 inches the experimental values at the centroid are appreciably higher than the calculated values. As in the case for beam 1, less weight should be given to the stress at the outside of the cover sheet in the evaluation of the stresses at the centroid. The stresses measured on the cover sheet near the flange rivets are much lower than the calculated values; whereas the agreement between calculated and experimental values for the corner angles is very good. These comparisons support the argument that the flange rivets between the root and station $7\frac{1}{2}$ inches did not transmit the forces as expected. The sum of the internal forces in the cover agrees with the external force within 2 percent at the root, 3 percent at station $2\frac{1}{2}$ inches, and 8 percent at station 5 inches. There were no signs of buckling of the cover sheet between rivets.

The better agreement of forces for beam 2 as compared with beam 1 is explained by the fact that measurements were taken on the corner-flange angle for beam 2; whereas the flange was assumed to have the stress indicated by the gage next to the flange rivets for beam 1.

<u>Measurements at high loads</u>.- Curves of applied load plotted against stress, for all individual measurements, showed that the stresses were not proportional to the loads in the higher regions: They were larger than calculated for 50 percent of the gages, equal for 35 percent, and lower for the remaining 15 percent. At present there is not enough information properly to explain this behavior. If a satisfactory explanation is sought, it should be remembered that the yield point of the material has been exceeded in several parts of the beam. Other factors that may help to explain the discrepancies are: the cover sheet buckled, with resulting loss of effectiveness, at a jack load of 10 kips; the rivets in the stringers might have

been inadequate for transmitting the required force from cover sheet to Z-stringer; and local bending in the stringers existed.

Ultimate Strength Tests

Failures of beams.—The failure in beam 1, which occurred at a jack load of 14,300 pounds with the distributed-load condition, is shown in figure 8. The corner flange failed first, followed by failure of the adjoining stringers. In order to prevent extensive damage of the specimen the jack load was released at an early sign of destruction. The stringer next to the flange was nevertheless torn near a rivet. The stringer along the longitudinal center line (stringer 9) was disjoined from the bulkhead at the root. This bulkhead suffered bad distortions at several places near the stringers; these distortions indicated the existence of large secondary bending forces. In the second beam the root bulkhead was attached to the stringers by a 1-by 1-by 1/8-inch steel angle.

Beam 2 failed at a jack load of 17,900 pounds with the distributed-load condition. Figure 9 shows the details the failure. As in beam 1 the flange was first to fail, followed by the failure of several adjoining stringers. Unlike beam 1, which failed gradually, beam 2 failed very suddenly. The corner-flange material was torn, as was the material in the three adjoining stringers. In contrast to the stringers of beam 1, all the stringers remained attached to the bulkheads.

Strength of corner flanges.— The calculated ultimate stress, in the flange at the root station, for beam 1 was 47,400 pounds per square inch; whereas 50,450 pounds per square inch was calculated for beam 2. These values are quite close to the column yield stress of 50,000 pounds per square inch (reference 3, fig. 5-1).

Strength of cover sheets.— If one Z-stringer and the width of cover sheet that accompanies it are isolated, the radius of gyration ρ of the section is found to be 0.350 inch. If the support given to the stringers by the bulkheads is assumed to be the equivalent of a pin end, the effective column length of the stringer is $L = 22$ inches. The resulting slenderness ratio, $L/\rho = 63$, is used to obtain the allowable column strength from the appropriate curve for 24S-T material (reference 3, fig. 5-1). The allowable column stress is 23,000 pounds per square inch.

At failure the maximum Mc/I stress was 24,400 pounds per square inch for beam 1 and 29,000 pounds per square inch for beam 2. These values may be interpreted to mean that the stringers were first to fail, with average fixity coefficients of 1.08 and 1.61, respectively. Inspection of the various parts of both beams under load, however, showed that failure first occurred in the corner flanges. The fallacy of the argument that the stringers failed by instability is obvious also because increasing the area of the corner flange can hardly be expected to increase the fixity coefficient developed by the stringers.

CONCLUSIONS

Shear lag is an integrated effect over the structure and yielding in local areas of the cover (such as the corner flanges at the root) should have little influence on the total effect. The shear-lag theory should therefore be valid for predicting the bending stresses in the cover of a box beam for high loads as well as for loads that produce stresses below the elastic limit of the material.

The results of the tests described in this report tend to confirm the foregoing theoretical conclusion. The experimental corner-flange stresses, which are the highest stresses in the cover, agree fairly well with the calculated stresses at all loads. The ultimate stress developed by the corner flange, where failure started, was found to be quite close to the column yield stress.

Langley Memorial Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va.

APPENDIX

ANALYSIS OF MULTISTRINGER BEAM 1 BY THE SUBSTITUTE SINGLE-STRINGER METHOD AND THE RECURRENCE FORMULA

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Idealization of cross section.- As the first step in the analysis of beam 1 the idealized cross section is found. All effective areas are considered to be concentrated in points that are joined by a fictitious shear-carrying cover sheet (fig. 2). The effective width w of cover sheet for the stringers is taken as one-half the stringer spacing b_1; no effective width is taken for the flange. The entire web is assumed to be effective in bending; one-sixth the area of the web is therefore considered to be concentrated at the flange.

The area of the idealized flange A_F consists of the following parts:

	(sq in.)
Corner angle	0.0848
Cover sheet, from rivet line to free edge (0.25 × 0.0422)	.0106
Equivalent of web (1/6 × 6.30 × 0.0806)	.0847
A_F	0.1801

The first stringer next to the flange is assumed to consist only of an effective width of sheet equal to 20t; the area of this stringer is

$$20 \times 0.0422 \times 0.0422 = 0.0356 \text{ sq in.}$$

Each of the seven adjoining stringers consists of a Z-stiffener and two strips of sheet each one-half as wide as b_1. The area of the idealized stringer is

$$A_{st} = 0.0664 + \left(2 \times \frac{2.125}{2} \times 0.0422\right) = 0.1560 \text{ sq in.}$$

The stringer at the center line has one-half this area, or 0.0780 sq in. The total area of the longitudinal is

$$A_L = 0.0356 + 7.5(0.1560) = 1.206 \text{ sq in.}$$

The analysis of this multistringer beam is made by the substitute single-stringer method and the recurrence formula.

First approximation of substitute single-stringer structure.- The basic data for this beam appear in table I. As the first approximation, all the stringers that are included in A_L are combined into a single longitudinal, which is located at the centroid of the stringers.

$$b_S = \frac{(7\times0.156\times4\times2.125) + (0.078\times8\times2.125)}{1.206} = 8.82 \text{ in.}$$

Thus, the centroid is found to be 8.82 inches from the flange. According to definition, this distance is the substitute width b_S that is used as the first approximation in the calculations.

From the assumption that the ratio $G/E = 0.385$, the shear-lag parameter K is found as follows:

$$K^2 = \frac{Gt}{Eb_S}\left(\frac{1}{A_F} + \frac{1}{A_L}\right)$$

$$= \frac{0.385 \times 0.0422}{8.82}\left(\frac{1}{0.180} + \frac{1}{1.206}\right)$$

$$= 0.00184 \times 6.388$$

$$K^2 = 0.01177$$

whence

$$K = 0.1081$$

The beam is divided into bays along the span, the web shear being constant throughout each bay. Stations are taken at each point of application of load; four equal bays result. The spanwise variation of the corner flange stress σ_F is known to be large near the root. The root bay is therefore subdivided into two bays and another station appears. Unequal bays were used so that a station at which strain measurements were taken would result. This spacing of stations allowed comparison of the group of experimental values with the calculated curve of distribution without interpolation between stations.

Calculations for distributed load; jack load = 8000 lb.- Table II shows the detailed calculations of the coefficients that are used in the recurrence formula. Because G and t are constant throughout the beam, the common factor Gt has been omitted from all coefficients, as an arithmetical simplification.

When the coefficients that were computed in table II are used, the equations for the X-forces are written according to equation (5) of reference 1. The boundary conditions are $X = 0$ at the tip and $Y = 0$ in the foundation bay, or $X_0 = 0$ and $Y_{r+1} = 0$. The equations are

$$- X_1(0.1100 + 0.1100) + X_2(0.0202) = -69 + 138$$

$$X_1(0.0202) - X_2(0.1100 + 0.1100) + X_3(0.0202) = -138 + 207$$

$$X_2(0.0202) - X_3(0.1100 + 0.1180) + X_4(0.0471) = -207 + 276$$

$$X_3(0.0471) - X_4(0.1180 + 0.1615) + X_5(0.1200) = -276 + 276$$

$$X_4(0.1200) - X_5(0.1615) = -276$$

These equations were solved and the computation of the stresses in the substitute single-stringer beam is given in table III.

Second approximation of substitute single-stringer structure. - For the first approximation the substitute width of beam was taken as the distance from the flange to the centroid of the longitudinals. In the second approximation b_S is the distance from the flange to the resultant internal force in the longitudinals. Computation of the new values of b_S, however, is not necessary; instead, a correction to X may be found from figure 15 of reference 1 and applied directly. After the second approximation the factor $\sqrt{2\,[1 - (y_L/b)]}$ differs by only 1 percent from the corresponding factor in the first approximation. The second approximation is therefore taken as final. (See table IV.)

Calculation of chordwise stress distribution.- After the spanwise distribution of stresses in the substitute single-stringer beam has been found, the chordwise distribution is calculated. The computations are shown in table V. The final values of stresses are the "corrected values."

The calculations are repeated to find the chordwise distribution at the other stations. If a cross section intermediate to the original stations is being considered, the bay in question is treated as a free panel, and the X-forces at the cross section within this bay are calculated by the following interpolation formula:

$$X_x = X_{n-1}\frac{\sinh K(L-x)}{\sinh KL} + X_n \frac{\sinh Kx}{\sinh KL}$$

where x is measured from station $(n-1)$ and L is the length of the bay.

Calculation of stresses at any load.- It will be recalled that the preceding calculations were made for a concentrated load of 500 pounds at each bulkhead. In order to find the stress distribution at any load, the proportional part of the stresses calculated for 500 pounds is taken. This simple linear relationship exists as long as the stresses do not cause buckling of the cover sheet.

After the critical buckling stress of the cover sheet is exceeded, the sheet continues to become less effective. In calculations for the stresses at those cross sections that have suffered such a loss of effective area, the revised section properties must be determined.

The effective width of cover sheet is calculated by Marguerre's approximation formula

$$2w = b_1 \sqrt[3]{\sigma_{cr}/\sigma}$$

where

w effective width of sheet

b_1 distance between rivet lines of adjoining stringers

σ_{cr} compressive buckling stress

σ average stress in stringers

The critical buckling stress of the cover sheet was found by assuming each panel to be simply supported at the Z-stringers and to have an aspect ratio of infinity (reference 4, p. 605). These assumptions gave a value of σ_{cr} = 15,000 pounds per square inch.

For beam 1 with distributed load the cover sheet was found to be 91 percent effective in resisting compressive forces. The final stresses at a jack load of 14,000 pounds are shown in fig. 4. The cover sheet was 88 percent effective for beam 2.

When a beam is analyzed at stresses beyond the buckling stress of the cover, it is usually necessary to revise only the original Mc/I calculations by taking into account the reduction in the effective area of the cover. Although the coefficients used in the recurrence formula may be changed by these revised areas, the X-force at any station depends, for the most part, upon the average conditions for the entire beam (that is, upon the average value of K). The individual value of K at the station in question has little influence on the X-force. Therefore, the X-forces as obtained from the calculations for low loads (entire area of cover effective) are used. At any station large changes in effective areas result in rather small changes in X-forces and still smaller changes in total stress. Thus, the need for repeating the X-force calculations is precluded. The stresses due to the X-forces are changed for the stringers only, because the flange suffers no loss in effectiveness. From the new values of σ_L the calculations for the chordwise distributions of stress are made as before.

REFERENCES

1. Kuhn, Paul, and Chiarito, Patrick T.: Shear Lag in Box Beams — Methods of Analysis and Experimental Investigations. NACA Rep. No. 739, 1942.

2. Aitchison, C. S., and Tuckerman, L. B.: The "Pack" Method for Compressive Tests of Thin Specimens of Materials Used in Thin-Wall Structures. NACA Rep. No. 649, 1939.

3. Anon.: Strength of Aircraft Elements. ANC-5, Army-Navy-Civil Committee on Aircraft Requirements. U.S. Govt. Printing Office, Oct. 1940.

4. Timoshenko, S.: Strength of Materials. Pt. II. D. Van Nostrand Co., Inc., 1930.

TABLE I.

BASIC DATA FOR ANALYSIS OF TEST BEAMS

Beam	A_F in root bay (sq in.)	A_F in typical bay (sq in.)	A_L (sq in.)	A_T of root bay (sq in.)	A_T of typical bay (sq in.)	t (in.)	h (in.)
1	0.180	0.180	1.206	1.386	1.386	0.0422	6.30
2	.276	.176	1.206	1.482	1.382	.0422	6.30

TABLE II.- COMPUTATION OF COEFFICIENTS FOR RECURRENCE FORMULA
(FIRST APPROXIMATION)

$$\left[\frac{1}{A_F} = 5.56;\ \frac{1}{A_L} = 0.828;\ \frac{A_L}{A_T} = 0.870;\ h = 6.30\text{ in.};\ K = 0.1081;\ \frac{G}{E} = 0.385;\ \text{jack load} = 8000\text{ lb}\right]$$

Bay	L	KL	tanh KL	sinh KL	p	q	S (lb)	S/h (lb/in.)	Y
1	22.0	2.38	0.983	5.356	0.1100	0.0202	500	79	69
2	22.0	2.38	.983	5.356	.1100	.0202	1000	159	138
3	22.0	2.38	.983	5.356	.1100	.0202	1500	238	207
4	14.5	1.57	.917	2.299	.1180	.0471	2000	318	276
5	7.5	.81	.670	.902	.1615	.1200	2000	318	276

TABLE III.- STRESSES IN SUBSTITUTE SINGLE-STRINGER BEAM
(FIRST APPROXIMATION)

$$\left[\frac{I}{y} = 8.27\text{ in.}^3\text{ (for outside fiber)};\ A_F = 0.180\text{ sq in.};\ A_L = 1.206\text{ sq in.};\ \text{jack load} = 8000\text{ lb}\right]$$

Station	x (in.)	M (lb-in.)	σ^P (lb/sq in.)	X (lb)	X/A_F (lb/sq in.)	σ_F (lb/sq in.)	X/A_L (lb/sq in.)	σ_L (lb/sq in.)	σ_L/σ_F	Yb
1	22.0	11,000	1,330	-347	-1,930	-600	-288	1,618	-2.70	1.40
2	44.0	33,000	3,995	-356	-1,975	2,020	-295	4,290	2.12	1.04
3	66.0	66,000	7,990	-118	-656	7,334	-98	8,088	1.10	.50
4	80.5	106,250	12,825	1049	5,825	18,650	870	11,955	.64	1.40
5	88.0	110,000	13,310	2488	13,820	27,130	2060	11,250	.42	2.45
									Total	6.79
									Average	1.36

From figure 15 of reference 1

$$\left(1 - \frac{y_L}{b}\right) = 0.452$$

$$\frac{1}{\sqrt{2\left(1 - \frac{y_L}{b}\right)}} = \frac{1}{\sqrt{0.904}} = 1.052 \text{ (for correcting K)}$$

TABLE IV.- STRESSES IN SUBSTITUTE SINGLE-STRINGER BEAM
(SECOND APPROXIMATION)

[Jack load = 8000 lb]

Station	σ^P (lb/sq in.)	X (lb)	X/A_F (lb/sq in.)	σ_F (lb/sq in.)	X/A_L (lb/sq in.)	σ_L (lb/sq in.)	σ_L/σ_F	Yb
1	1,330	-327	-1,815	-485	-271	1,601	-3.30	1.40
2	3,995	-337	-1,870	2,125	-279	4,274	2.01	1.03
3	7,990	-134	-745	7,245	-111	8,101	1.12	.55
4	12,825	963	5,350	18,175	798	12,027	.66	1.35
5	13,310	2379	13,200	26,510	1970	11,340	.43	2.30
							Total	6.63
							Average	1.33

From figure 15 of reference 1

$$\left(1 - \frac{y_L}{b}\right) = 0.438$$

$$\frac{1}{\sqrt{2\left(1 - \frac{y_L}{b}\right)}} = \frac{1}{\sqrt{0.876}} = 1.068$$

TABLE V. - COMPUTATION OF CHORDWISE STRESS DISTRIBUTION AT ROOT (STATION 5)

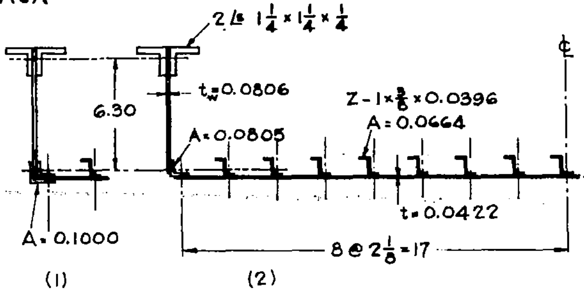
[Jack load = 8000 lb]

Stringer	y	Yy	cosh Yy	σ (lb/sq in.) (1)	A_{st} (sq in.)	σA_{st} (lb) (1)	σ (lb/sq in.) (2)	σA_{st} (lb) (2)
1	b	2.30	5.04	26,510	0.036	(955)	26,510	955
2	$\frac{7}{8}b$	2.01	3.81	20,000	.156	3,120	21,000	3,280
3	$\frac{3}{4}b$	1.73	2.90	15,250	.156	2,380	16,000	2,495
4	$\frac{5}{8}b$	1.44	2.22	11,675	.156	1,820	12,250	1,910
5	$\frac{1}{2}b$	1.15	1.74	9,150	.156	1,425	9,610	1,500
6	$\frac{3}{8}b$	.86	1.40	7,350	.156	1,145	7,730	1,205
7	$\frac{1}{4}b$	.58	1.17	6,150	.156	960	6,460	1,005
8	$\frac{1}{8}b$	.29	1.04	5,470	.156	855	5,750	898
℄ 9	0	0	1.00	5,250	.078	410	5,520	432
				Totals	1.206	12,115		13,680

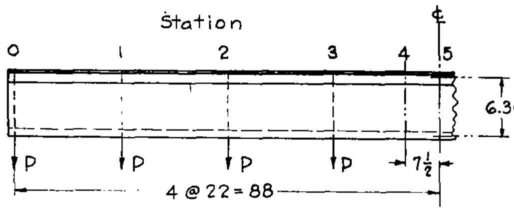
[1]Uncorrected values.
[2]Corrected values.

$$\sigma_L A_L = 11,340 \times 1.206 = 13,700$$
$$\frac{-955}{12,745}$$

$$\text{Correction factor} = \frac{12745}{12115} = 1.050$$



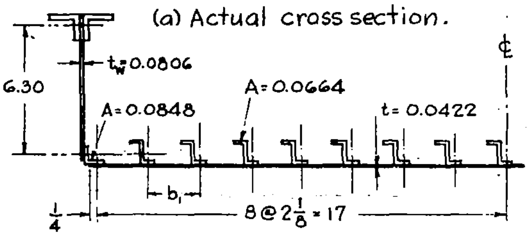
(a) Cross sections of beam 2 for (1) root bay (2) typical bay.



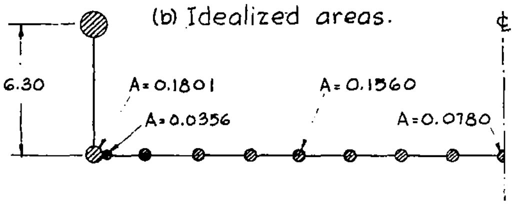
(b) Half-span of beams 1 and 2 with distributed load.

Figure 1.– Dimensions of test beams.

(a) Actual cross section.



(b) Idealized areas.



(c) Substitute single-stringer beam.

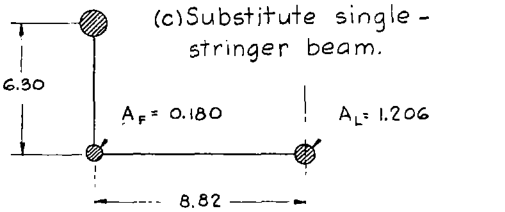
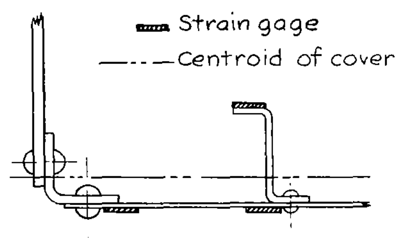
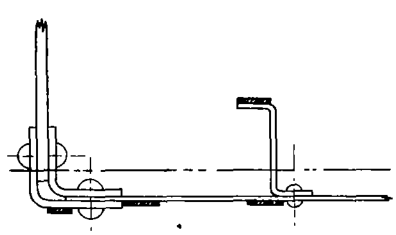


Figure 2.– Idealization of cross section of beam 1.



(a) Beam 1.



(b) Beam 2.

Figure 4.– Typical locations of electrical strain gages.

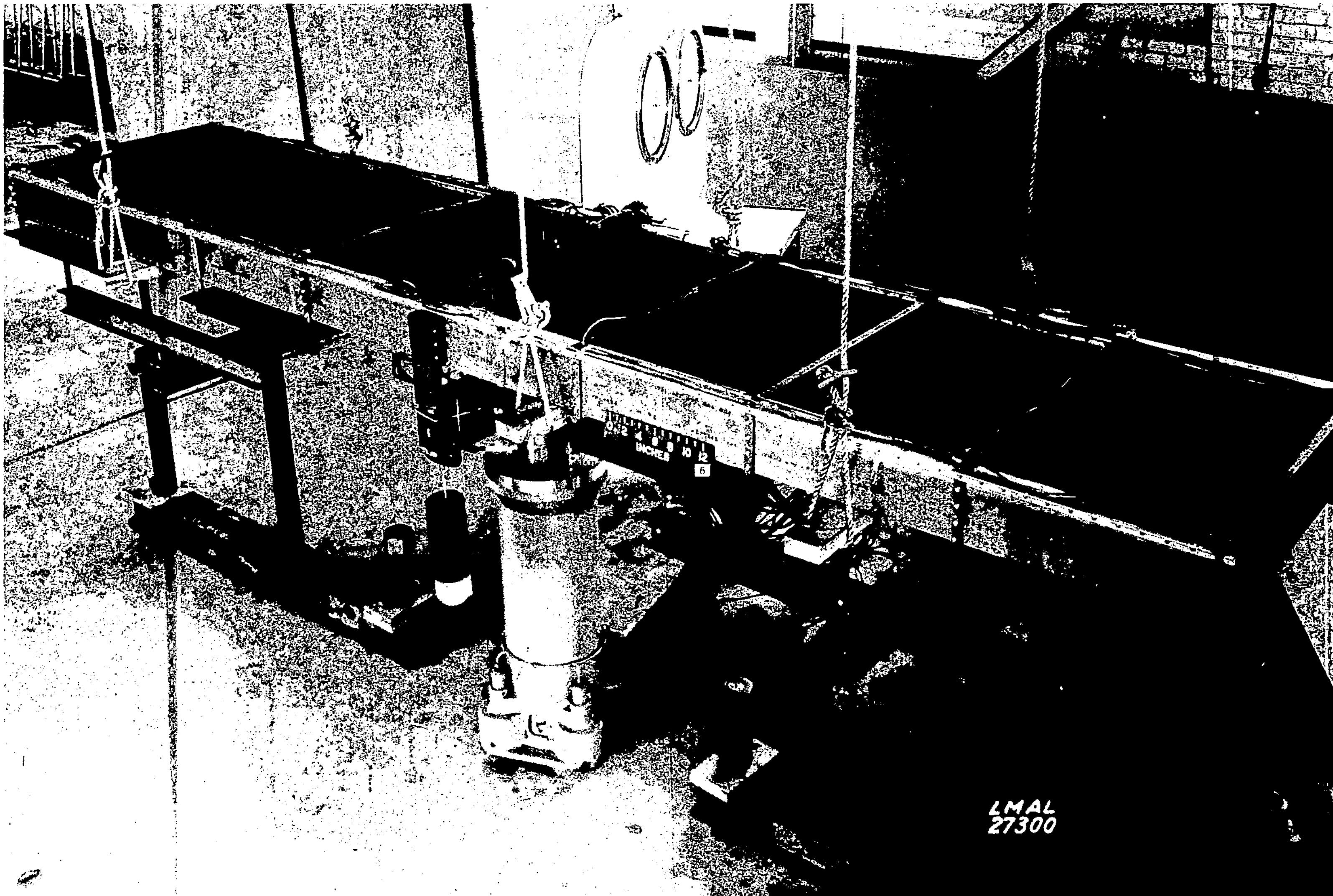
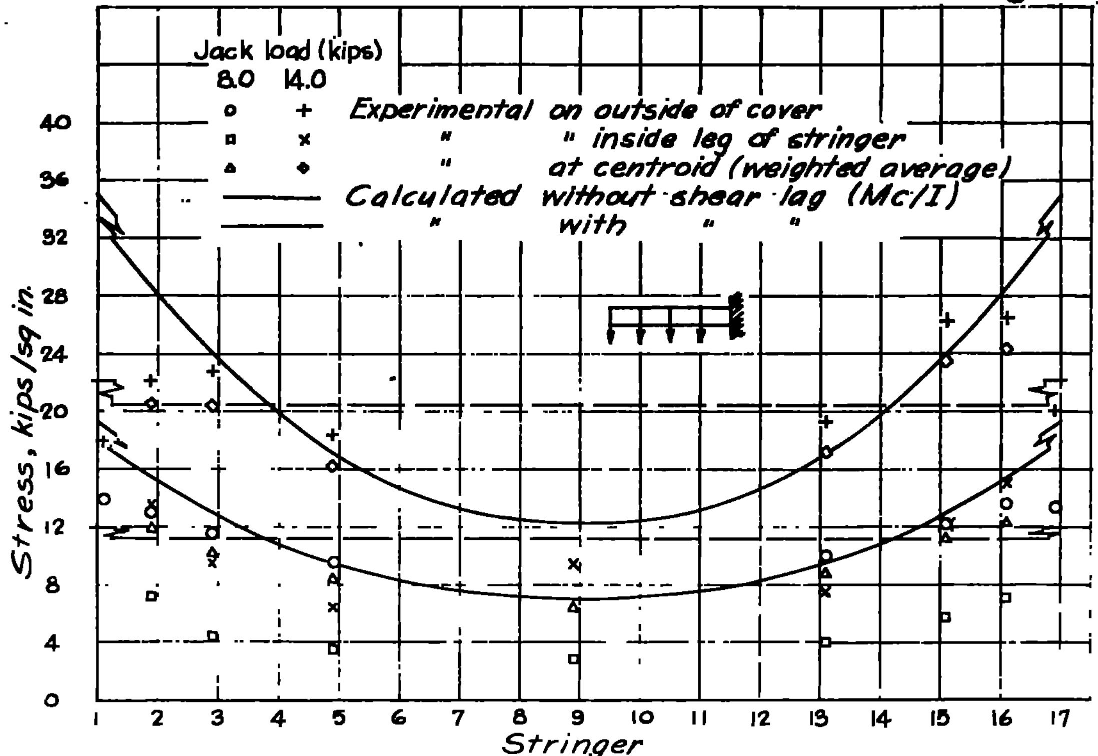


Figure 3.- General view of specimen and set-up.



(c) Station 5 inches.

Figure 5.– Concluded.

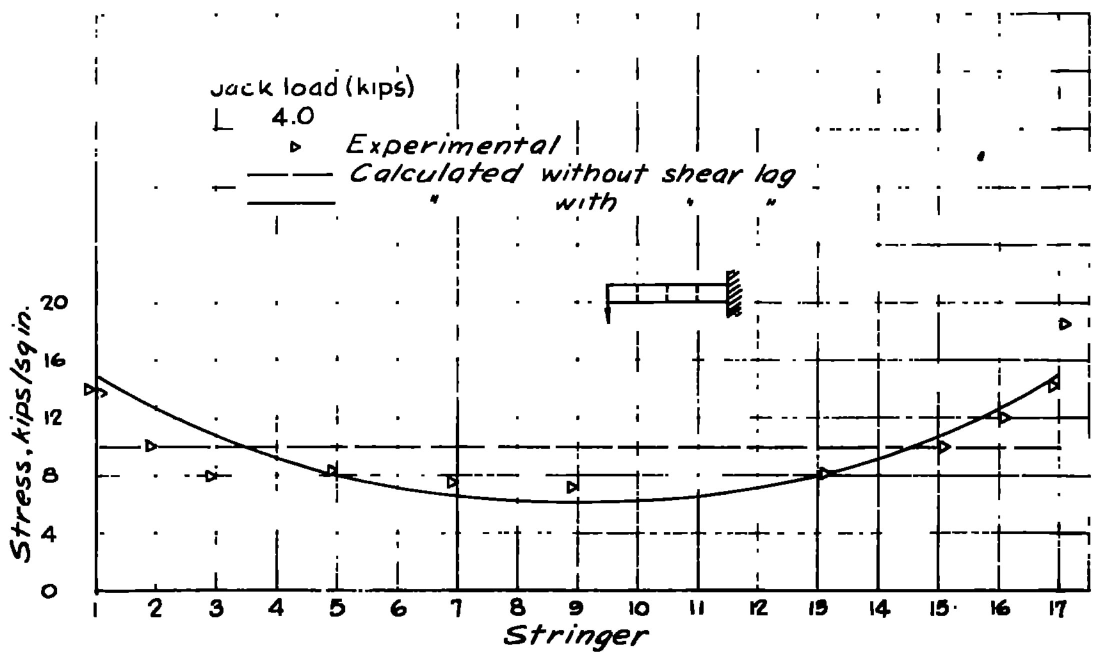
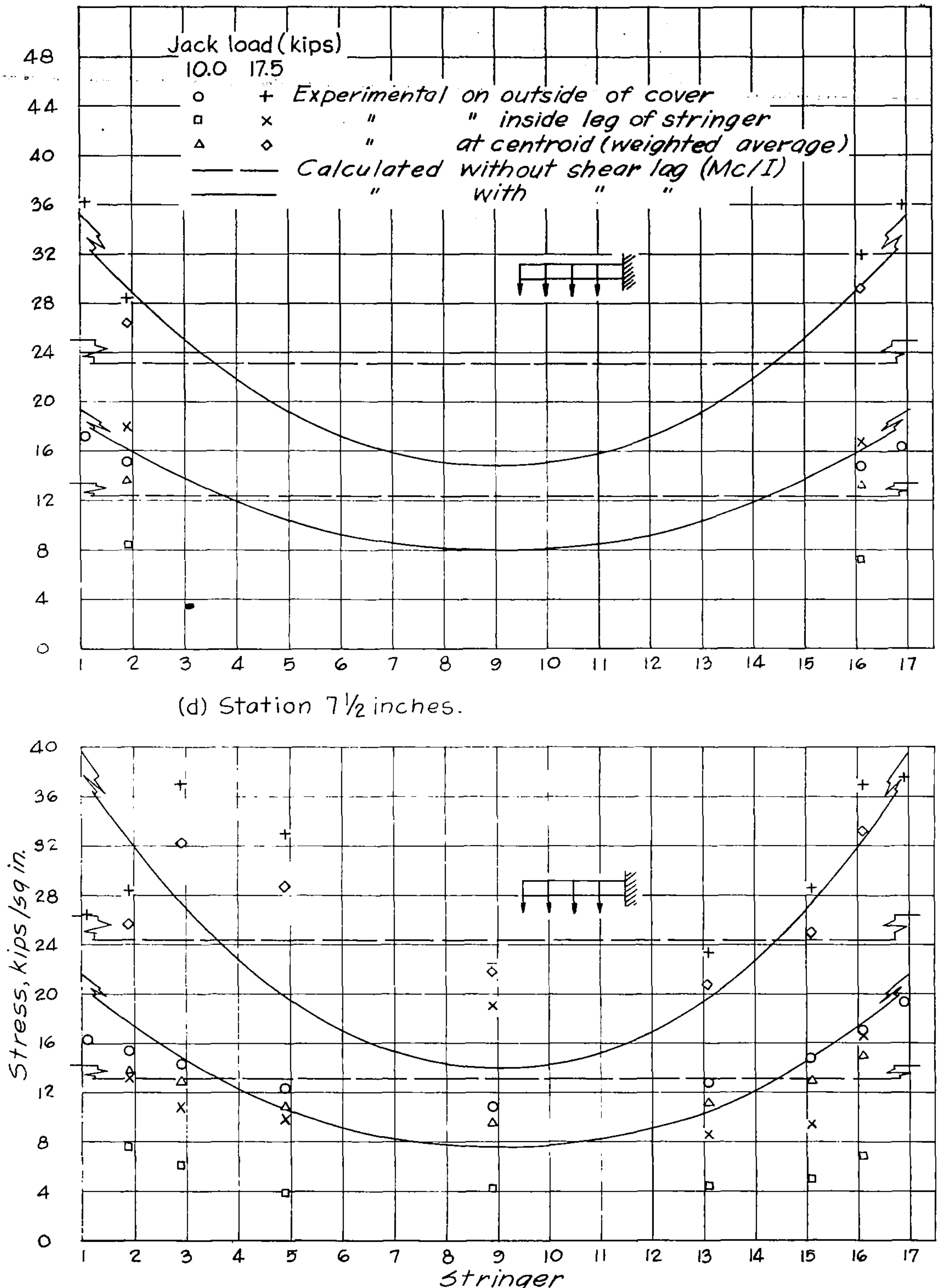


Figure 6.– Chordwise distribution of stresses at root station in beam 2 with tip load.



(d) Station 7½ inches.

(c) Station 5 inches.

Figure 7.– Concluded.

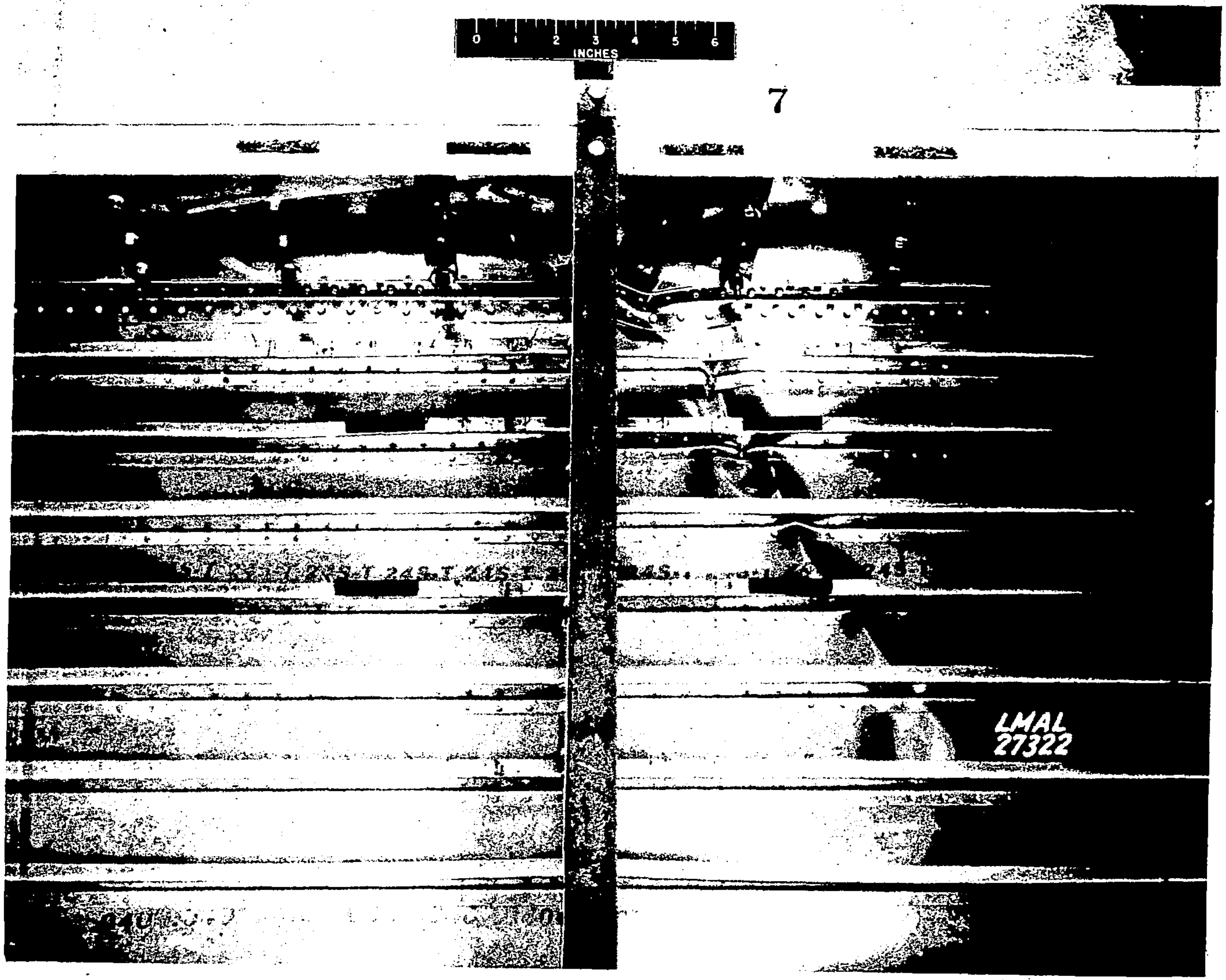


Figure 8.- Failure at root of beam 1.

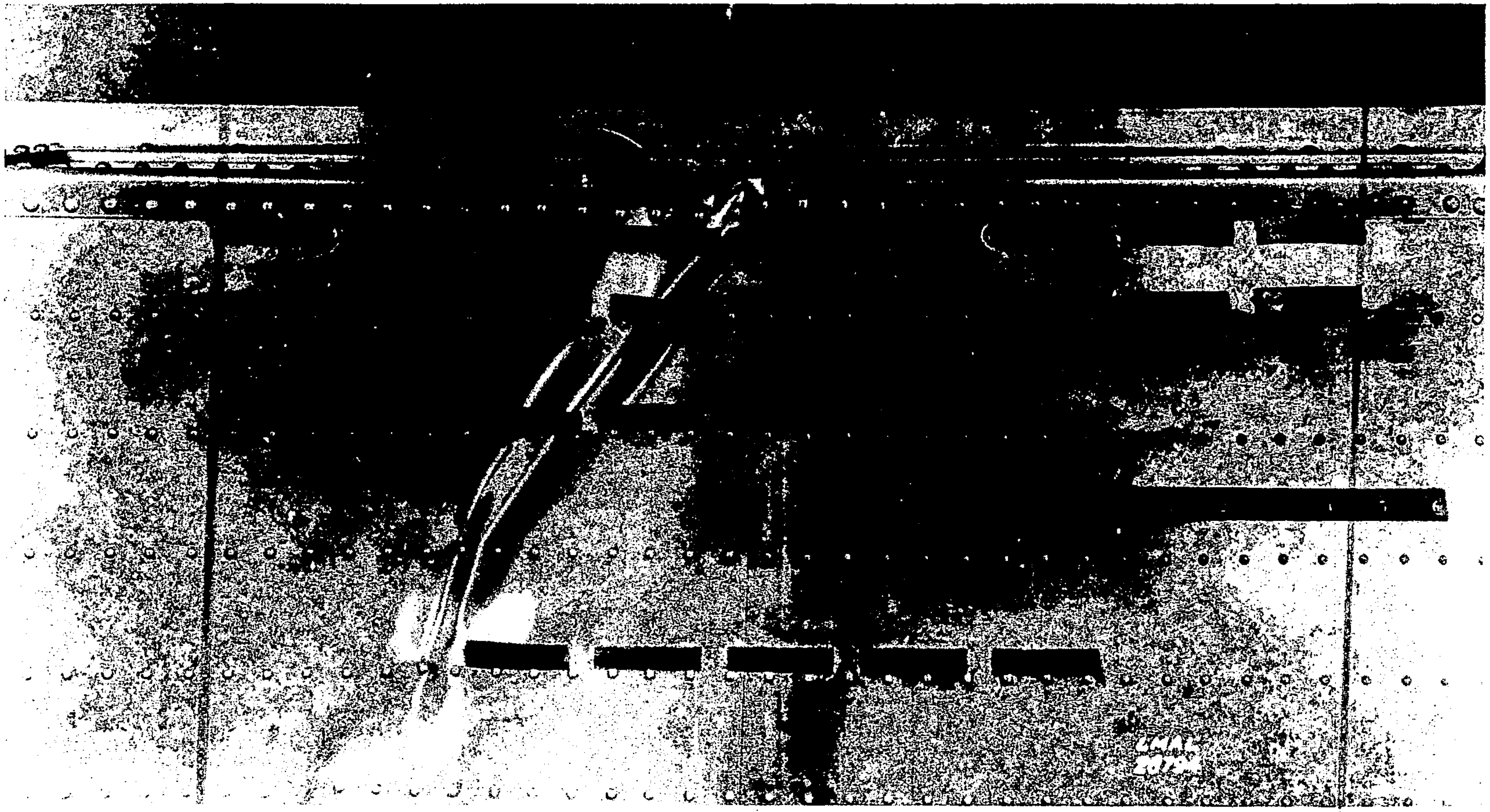


Figure 9.- Failure at root of beam 2.